Divide and Conquer:
1) divide input into pieces
2) solve subproblems on each piece
3) combine results

Examples:
- Quicksort
  1) split input into small/big (picks a pivot)
  2) sort each part
  3) rejoice!

- Mergesort
  1) split input list into two halves
  2) sort each part
  3) merge sorted halves

- Binary search
  1) split input into two halves
  2) search in appropriate half
  3) rejoice!

T(n) = O(n) + 2T(n/2) + O(n)

\[
T(n) = \begin{cases} 
1 & \text{if } n = 1, \\
T \left( \frac{n}{2} \right) + O(1) & \text{if } n \text{ is even}, \\
T \left( \frac{n-1}{2} \right) + O(1) & \text{if } n \text{ is odd}.
\end{cases}
\]

If steps to merge sort \( n \) items

\[
\log_b a = 1 = d
\]

\[
a = 2, b = 2, d = 0
\]

\[\text{middle case} \Rightarrow T(n) = \Theta(n \log n)\]

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\[\log_b a = \log_2 1 = 0 = d\]

\[\text{middle case} \Rightarrow T(n) = \Theta(n \log n)\]

\[= \Theta(n \log n)\]
Master Method: Solution to recurrence \( T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n) \)

is:

\[\Theta(n^{\log_b a}) \quad \text{if} \quad f(n) = O(n^{\log_b a - \varepsilon}) \quad d < \log_b a\]

\[\Theta(n^{\log_b a} \log n) \quad \text{if} \quad f(n) = \Theta(n^{\log_b a}) \quad d = \log_b a\]

\[\Theta(f(n)) \quad \text{if} \quad f(n) = \Omega\left(n^{\log_b a + \varepsilon}\right) \quad d > \log_b a\]

and \( a \cdot f\left(\frac{n}{b}\right) \leq c \cdot f(n) \) for some \( c < 1 \) and large \( n \).
Closest Pair: Given \( n \) points in the plane, determine the pair that are closest to each other.

**Brute Force:**
- Check each pair of points, keeping track of min distance.
- \( \Theta(n^2) \)

**Divide-and-Conquer:**
- Sort points by x-coord.
- Divide pb by x-coord.
- Find closest pair among left, right pts.
- Find closest of the closest pairs across split.

\[ T(n) = 0(1) + 2T\left(\frac{n}{2}\right) + O(1) + O(n^2) \]
\[ = 2T\left(\frac{n}{2}\right) + O(n^2) \]
\[ m=2 \quad b=2 \quad d=2 \]
\[ \log_b a = 1 \]

\[ d > \log_b a \]

(last case)

\[ T(n) = \Theta(n^2) \]

Need to do better in second step! (and will tomorrow...)

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