insert 1 8 32 22 31 33 34
Maintaining height

\[ \text{height}(m) \]

\[
\text{if } m \neq N/4 \\
\quad \text{return } 1 + \max(\text{height}(m, \text{left}), \text{height}(m, \text{right})) \quad \mathcal{O}(n) \\
\text{else} \\
\quad \text{return } 0
\]

recurrence for min size of AVL tree with height \( h \)

solution to that is \( \geq \text{Fib}_h \)

key fact: \( \text{Fib}_h \geq \phi^h \)

size (AVL tree of height \( h \)) \( \geq \phi^h \)

\[
\log \phi \leq \frac{\log n}{h} \\
\frac{\log n}{h} \leq \log \phi \\
h \leq \frac{\log n}{\log \phi} \\
h \leq \mathcal{O}(\log n)
\]
Invariants:

something always true about structure

part of invariant

AVL tree invariants

BST order property holds
balance property holds
n.pwrt.right = n

n.pwrt = n if n#head
Red-Black Trees

Binary Search Trees s.t.
1) each node has a color - red or black
2) root is black
3) all leaves are black
4) children of red nodes are black
5) all paths root $\rightarrow$ leaf have same # of black nodes