Amortized Time vs. Average Time

$O >$ Random inserts into standard BST $O(\log n)$ average per insert

$O \leq$

$\Theta =$

$O \leq$

Inserts into splay tree $O(\log n)$ amortized time per insert

$O \leq$

$P(n \text{ inserts take } o(n \log n) \text{ time}) = O$

*For every $c > 0$ there is an $n$ s.t. $P(n \text{ inserts take } \geq c \cdot n \log n \text{ time}) > 0$

* There is a $c > 0$ s.t. for all $c' > c$ there is an $n_0$ s.t. for all $n \geq n_0$ $P(n \text{ inserts take } \geq c' \cdot n \log n \text{ time}) = 0$
Amortized Analysis

Charge for each operation
1) cost of operation
2) cost of potential delay work

 amortized cost = actual cost + \( \Phi \)

Ex: Array Stack

<p>| ( \Phi(5) ) | 4. # elements in 2nd half |</p>
<table>
<thead>
<tr>
<th>total actual work</th>
<th>potential</th>
<th>total income</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>5 7</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>5 12</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>5 17</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>5 22</td>
</tr>
<tr>
<td>19</td>
<td>4</td>
<td>5 27</td>
</tr>
<tr>
<td>22</td>
<td>16</td>
<td>5 35</td>
</tr>
<tr>
<td>39</td>
<td>4</td>
<td>5 47</td>
</tr>
</tbody>
</table>

Each item pays for itself to be added. Itself to be copied on next resize. Corresponding item in 1st half to be copied. 2 empty spaces on next resize.

\[
\text{amortized cost} = \text{actual cost} + (\Phi(s') - \Phi(s))
\]

Case 1: add in 1st half
- actual cost = 1
- potential before = 0
- potential after = 0

Case 2: add in 2nd half, no resize
- \( 5 \geq 1 + 4 \)

Case 3: add \& resize
- \( 5 \geq 2n + 1 + \left( 4 - 4 \cdot \frac{5}{2} \right) \)
- \( = 2n + 1 + 4 - 2n \)
- \( = 5 \)