**Skip Lists**

**FIND (x)**

```
level ← highest
curr ← head
while level ≥ 0
  while curr.next[level] ≤ x
    curr ← curr.next[level]
  end
  last(level) ← curr
  level ← level - 1
  if last(1).next[1] = x
    return T
  end
end
return F
```

- number of nodes at level 1 = n
- \( E[\text{number of nodes at level 2}] = \frac{n}{2} \)
- \( E[\text{number of nodes at level i}] = \frac{n}{2^{i-1}} \)

\( E[2^n] = E[Y_1 + \cdots + Y_n] = E(X_1) + \cdots + E(X_n) = n \cdot E[X_1] = n \cdot \frac{1}{2} = \frac{n}{2} \)

\( E[X_j] = \sum_{i=1}^{n} P(\text{node j is at level i}) \)

\( = \sum_{i=1}^{n} P(\text{roll } i \text{ at level i}) \)

\( = 1 \cdot P(\text{roll } 1 + \cdots + \text{roll } 6) \)

\( = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6} \)

\( = \left( \frac{1}{6} + \frac{2}{6} + \cdots + \frac{6}{6} \right) \cdot \frac{1}{6} \)

\( = \frac{21}{6} \cdot \frac{1}{6} \)
\[
E[Y] = E[X_1 + X_2] = E[X_1] + E[X_2] = 3\frac{1}{2} + 3\frac{1}{2} = 7
\]

\[
E[Y] = (1 + 2 + \cdots + 6) \cdot \frac{1}{6}
\]

\[
= \frac{36}{6} = 3\frac{1}{2}
\]

\[
y = \text{outcome of rolling 2 dice}
\]

\[
y = \text{outcome of rolling 2 for 6-sided die}
\]

\[
E[Y] = \sum_{i=2}^{6} i \cdot P(\text{roll is } i)
\]

\[
= 2 \cdot P(\text{roll is 2}) + 3 \cdot P(\text{roll is 3})
\]

\[
P(1) = P(2) + P(3) \cdot P(1)
\]

\[
\frac{2}{36}
\]