\( J_6, m_5, L, L, K_6, G, J_6, C, K_6 \)

Not simple

\[ \{ G_6, J_6 \} \]
\[ \{ J_6, G_6 \} \]

(J6, GR) = (J6, GR)

Graph: represents things and relationships between them

\( \text{people} \downarrow \text{relationship} \downarrow \text{size-2 subsets (undirected)} \)

path: sequence of vertices \( V_0, V_1, \ldots, V_k \) s.t. \( \{ V_0, V_1 \}, \{ V_1, V_2 \}, \ldots, \{ V_{k-1}, V_k \} \) are edges in the graph

simple path: doesn't repeat vertices

cycle: a path with \( V_0 = V_k \)

simple cycle: a cycle w/ no unnecessary repeats

6 degrees of separation:

for any two people, is there a path of length \( \leq 6 \) between them

J6, m5, Kk, K6, J6 cycle
**Connected**

Connected: for any two vertices $u, v$, there is some path $u \to v$

Connected component: maximal subset of vertices $C$ s.t.
for every $u, v \in C$ there is a path $u \to v$

Connected $\Rightarrow$ entire graph is 1 connected component

$V = \text{NESCAC football teams}$

$E = \text{who played whom in 1st two weeks of 2014}$

Directed graph: edges are ordered pairs $(u, v) \neq (v, u)$

2 connected components

Strongly connected: for every pair of vertices $u, v$ $u \to v \text{ and } v \to u$

Strongly connected component: maximal subset so that every pair connected in both ways

Component graph: set of edges $(u, v) \in E \to u \text{ lost to } v$

Mid lost to Ambros

Were the Patriots the best team in the NFL last year?

Can I make the case that any $X > Y$

If $X$ is the graph strongly connected

$\Rightarrow$ not simple

Def $\Rightarrow$ not simple

$\Rightarrow$ 0

Weighted graph: graph w/ weight function $w : E \to \mathbb{R}$
weight of a path: sum of weights of edges on path

(u,v) means v beat u
\( w(u,v) \) = margin of victory

Amherst beat Bowdoin by 63

"shortest" path = min possible weight

Google maps: vertices = intersections
edges = roads between locations
weights = average travel time
int foo(int n, int c) {
    if (n == c) {
        return 0;
    }
    int i = 1;
    while (i < n) {
        if (i % c == 3) {
            if (n % 2 == 1) {
                return 0;
            }
        }
        i++;
    }
}

vertices: lines of code
edges: control flow
(free) tree: undirected, acyclic, connected graph

TFAE: 1) $G$ is a free tree (acyclic and connected)

2) $\forall u,v \in G.V$, $G$ has a unique simple path $u \rightarrow v$

3) $G$ is connected, but removing any edge disconnects $G$

4) $G$ is connected and $|E| = |V| - 1$

5) $G$ is acyclic and $|E| = |V| - 1$

6) $G$ is acyclic but adding any missing edge results in a cycle
rooted tree, ancestor, descendant,...
Graph Representation

Adjacency Matrix

\[
\begin{align*}
A & \quad A & \quad W & \quad M & \quad H & \quad C \\
A & \quad F & \quad F & \quad F & \quad F & \quad F \\
W & \quad T & \quad E & \quad T & \quad F & \quad F \\
M & \quad T & \quad E & \quad E & \quad F & \quad F \\
H & \quad F & \quad F & \quad F & \quad F & \quad F \\
C & \quad F & \quad F & \quad F & \quad F & \quad F \\
\end{align*}
\]

Adjacency List

A : A
W : A M
M : A
H : A M C